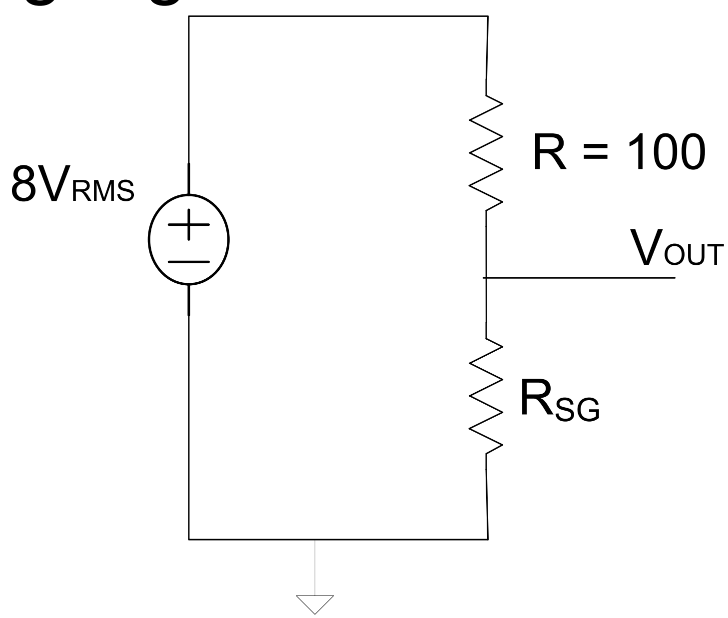
# EE 230

## Lecture 3

## Background Materials
## Transfer Functions

# Quiz 2

A strain gauge with a gauge factor of 2 and an unstrained resistance is used in the following circuit. Determine the change in output voltage due to a strain of ε = .0001. Assume the unstrained gauge resistance is 100Ω.



# And the number is ?

1

3

8

5

4

2

6

9

7

# And the number is ?

1 3 8

5 4

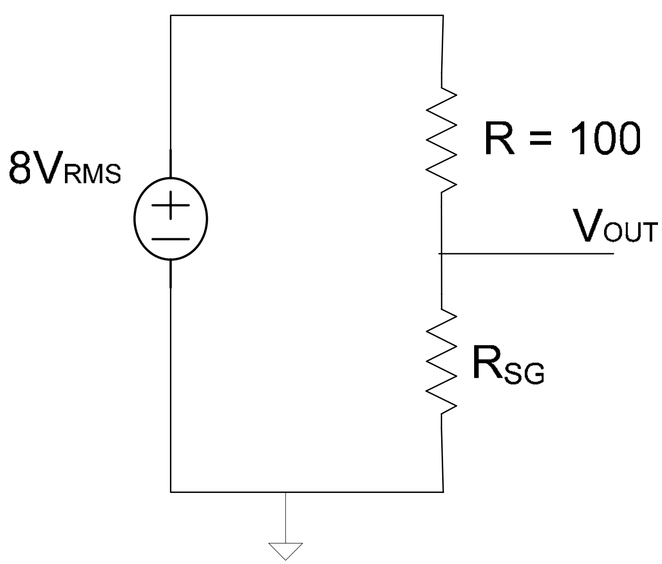
2 **4** 6

9 7

# Quiz 2

Solution:

A strain gauge with a gauge factor of 2 and an unstrained resistance is used in the following circuit. Determine the change in output voltage due to a strain of ε = .0001. Assume the unstrained gauge resistance is 100Ω.




$$\Delta R / R = \varepsilon \ \mathrm{GF} \cong 0.0002$$

$$V_{ON} = \frac{100}{100+100} 8\mathrm{V_{RMS}} = 4\mathrm{V_{RMS}}$$

$$\mathrm{R_{ST}} = \mathrm{R} + \Delta\mathrm{R} = \mathrm{R}(1.0002) = 100.02\Omega$$

$$V_{OST} = \frac{100.02}{100+100.02} 8\mathrm{V_{RMS}} = 4.0004\mathrm{V_{RMS}}$$

$$\Delta V_{OUT} = V_{OST} - V_{ON} = (4-4.0004)\mathrm{V_{RMS}} = 400\mu\mathrm{V_{RMS}}$$

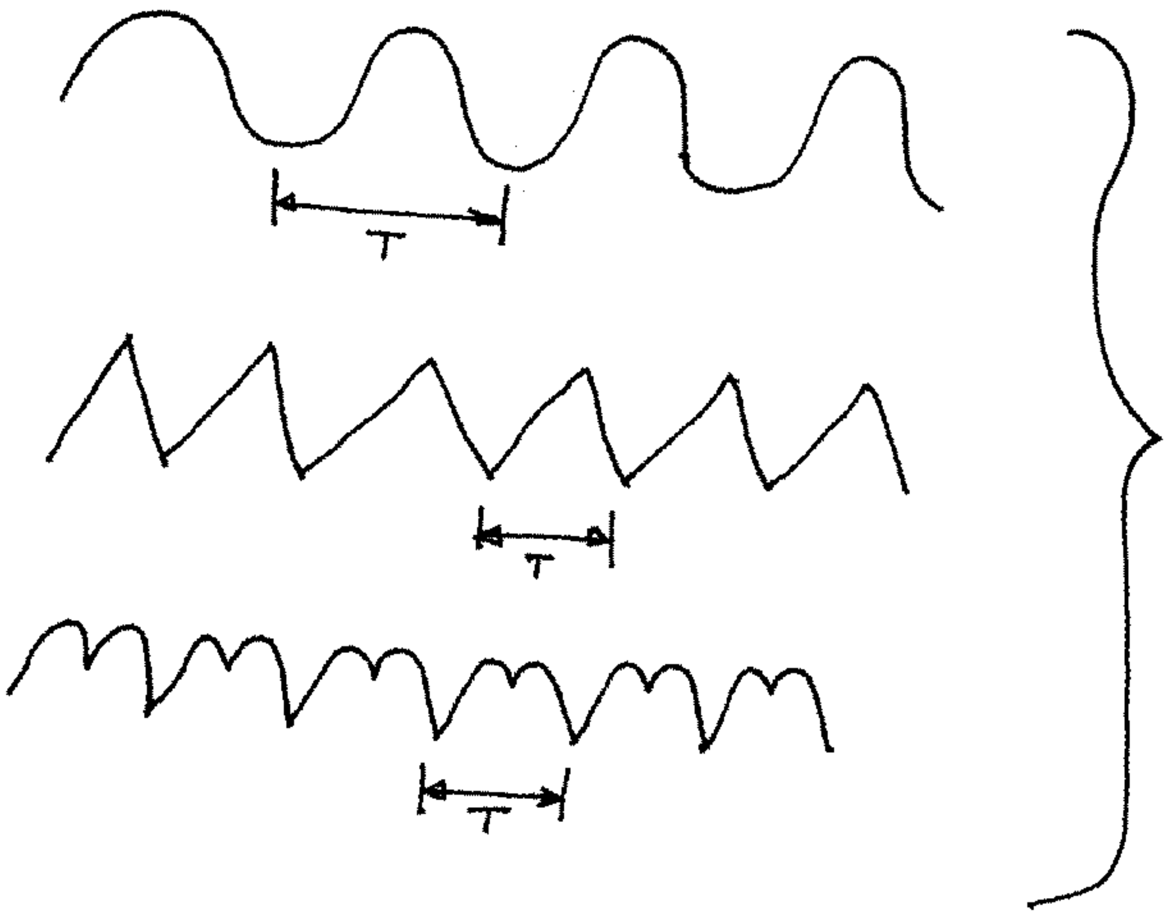
# Review from Last Time

Strain Gauges are one transducer that provides very small changes in electrical variables in the presence of strain

Load Cells combine strain gauges for measuring force – signals conditioning is usually included in the load cell

Review from Last Time

Many continuous-time signals nearly periodic




$$V_o(t+kT) = V_o(t)$$

Review from last

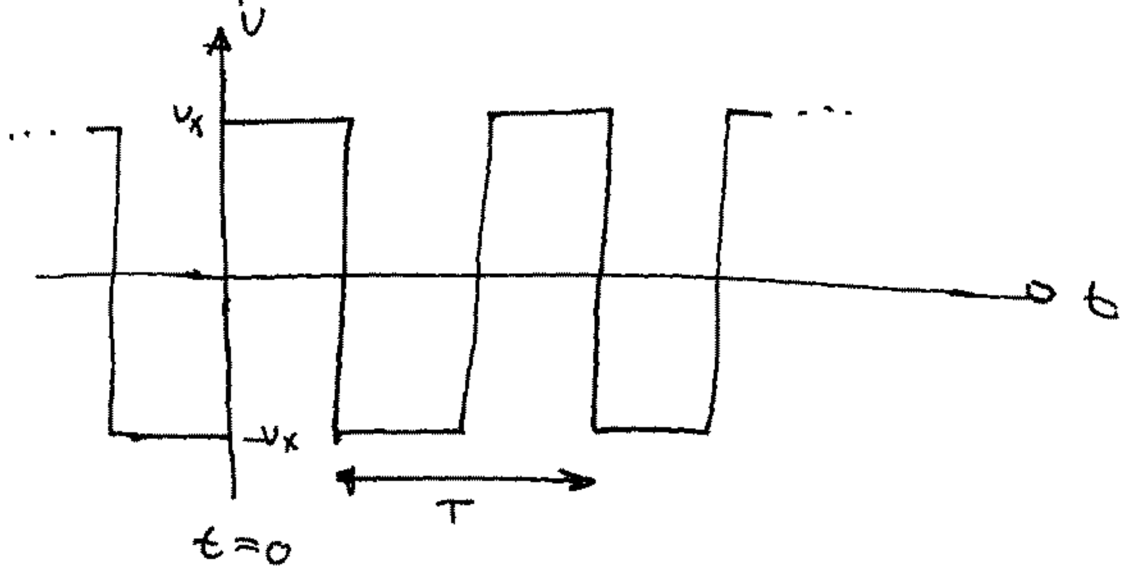
Theorem: If $f(t)$ is periodic with period $T$, then $f(t)$ can be expressed as

$$f(t) = \sum_{k=0}^{\infty} A_k \sin(k\omega t + \theta_k)$$

where $A_k$ & $\theta_k$ are constants and $\omega = \frac{2\pi}{T} = 2\pi f$

- This is termed the Fourier Series Representation
- $\langle A_k, \theta_k \rangle_{k=0}^{\infty}$ termed frequency spectrum of $f(t)$
- $f(t) \longleftrightarrow F(\omega)$ represent a transform pair

Example:

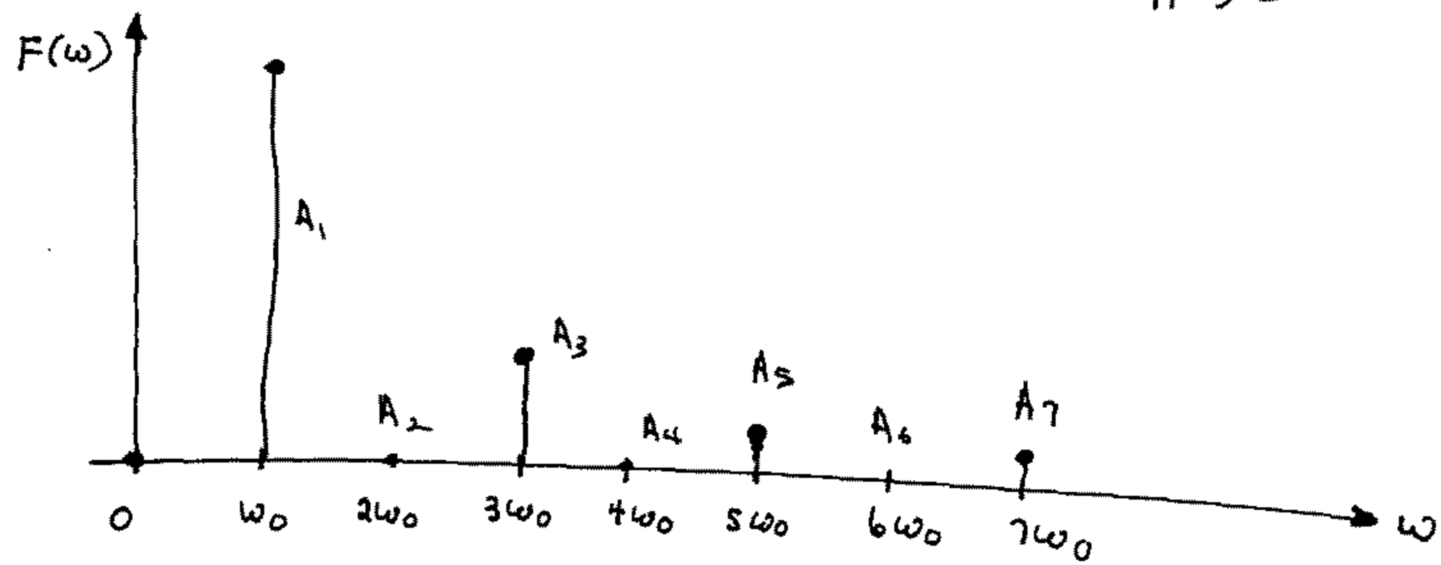



$$V_{sq}(t) = \frac{4V_x}{\pi}\left(\sin\omega_0 t + \frac{1}{3}\sin 3\omega_0 t + \frac{1}{5}\sin 5\omega_0 t + \cdots\right)$$

$$V_{sq}(t) = \frac{4V_x}{\pi}\sum_{\substack{k=1\\ k\ \text{odd}}}^{\infty}\frac{\sin(k\omega_0 t)}{k}$$

where $\omega_0 = \dfrac{2\pi}{T}$

$$F(\omega) = \frac{4V_x}{\pi},\ 0,\ \left(\frac{4V_x}{\pi}\right)\frac{1}{3},\ 0,\ \left(\frac{4V_x}{\pi}\right)\frac{1}{5},\ 0,\ \cdots$$




$A_1$ termed fundamental

$A_2$ termed second harmonic

$\vdots$

$A_k$ termed the kth harmonic

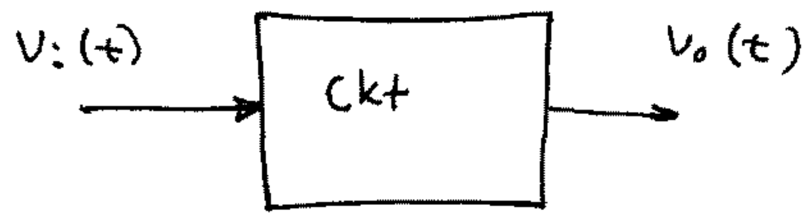
- Nonperiodic Signals Can Also Be Represented in the frequency domain
- Fourier Transform Used for this purpose
- Discrete Time Signals Can Also Be Represented in the frequency domain
- Discrete Fourier Transform (DFT) used for this purpose

- Often interested in knowing how sinusoidal signals propagate through a circuit

- Often design circuits so that sinusoidal signals will propagate through the circuit in a predetermined way

- This is the major reason a strong emphasis on analyzing circuits with sinusoidal excitations was made in EE 201

# Linearity




- A circuit is linear if

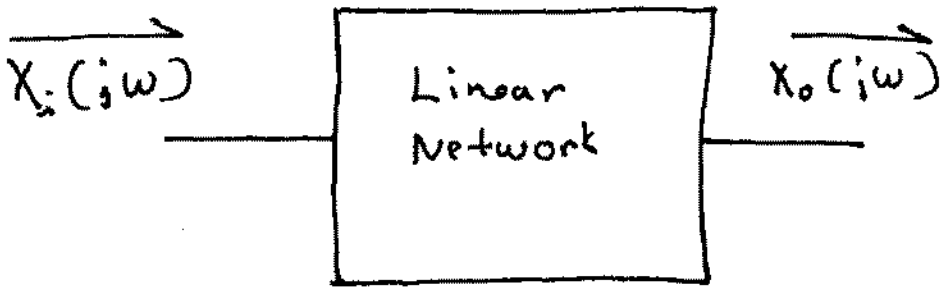
$$V_o\,(a_1 V_1 + a_2 V_2) = a_1 V_o(V_1) + a_2 V_o(V_2)$$

for all $V_1, V_2$ and all $a_1, a_2$

- If a circuit is linear, the dc transfer characteristics is a straight line

- If the dc transfer characteristics are not a straight line, the circuit is not linear

# Properties of Linear Networks




$$\frac{\overrightarrow{X_o(j\omega)}}{\overrightarrow{X_i(j\omega)}} = T_p(j\omega)$$

$T_p(j\omega)$ is called the <u>phasor transfer function</u>

$$T_p(j\omega) = |T_p(j\omega)|\, e^{j(\arg(T(j\omega)))}$$

$$= |T_p(j\omega)| e^{j\Theta} \qquad \Theta = \arg(T(j\omega))$$

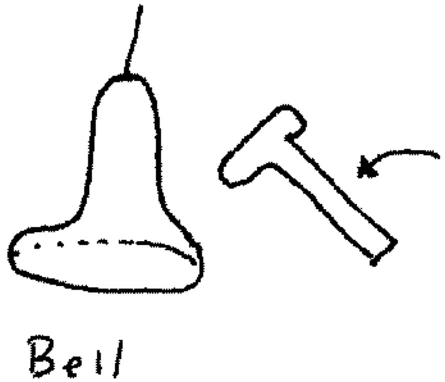
If a sinusoidal input is applied to a linear system, no harmonics are present in the output

If a sinusoidal input is applied to a nonlinear system, harmonic components often appear in the output

If a sinusoidal input is applied to a system and harmonic components appear in the output, the system is nonlinear.

The introduction of harmonics by a nonlinear system introduces distortion <u>and</u> distortion (even small amounts) is very undesirable in many applications

Example:




- Striking a bell results in a nearly sinusoidal waveform that sounds pleasurable
- If the sinusoidal output were altered in an amplifier or by a fault in the bell, the sound would usually be very objectionable

# Total Harmonic Distortion

Consider a periodic function with zero average value

$$f(t) = \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k)$$

If $f(t)$ is a voltage driving a resistive $1\Omega$ Load,
then $\quad P(t) = f^2(t)$

Define $P_1$ to be the power of the fundamental.

$$P_1 = \frac{A_1^2}{2}$$

It can be shown that

$$P_f = \frac{\sum_{k=1}^{\infty} A_k^2}{2}$$

$$P_{HARMON} = \frac{\sum_{k=2}^{\infty} A_k^2}{2}$$

$$THD = \frac{P_{HARMON}}{P_1}$$

$$\therefore \quad THD = \frac{\sum_{k=2}^{\infty} A_k^2}{A_1^2}$$

Often Expressed in dB

$$THD_{dB} = 10\log_{10}(THD)$$

# Total harmonic distortion

From Wikipedia, the free encyclopedia

The **total harmonic distortion**, or **THD**, of a signal is a measurement of the harmonic distortion present and is defined as the ratio of the sum of the powers of all harmonic components to the power of the fundamental.

## Explanation

In most cases, the transfer function of a system is linear and time-invariant. When a signal passes through a non-linear device, additional content is added at the harmonics of the original frequencies. THD is a measurement of the extent of that distortion.

The measurement is most commonly the ratio of the sum of the powers of all harmonic frequencies *above* the fundamental frequency to the power of the fundamental:

$$\mathrm{THD} = \frac{\sum \text{harmonic powers}}{\text{fundamental frequency power}} = \frac{P_2 + P_3 + P_4 + \cdots + P_n}{P_1}$$

Other calculations for amplitudes, voltages, currents, and so forth are equivalent. For a voltage signal, for instance, the ratio of RMS voltages is equivalent to the power ratio:

$$\mathrm{THD} = \frac{\sqrt{V_2^2 + V_3^2 + V_4^2 + \cdots + V_n^2}}{V_1}$$

In this calculation, $V_n$ means the RMS voltage of harmonic $n$.

Other definitions may be used. A measurement must specify how it was measured. Measurements for calculating the THD are made at the output of a device under specified conditions. The THD is usually expressed in percent as distortion factor or in dB as distortion attenuation. A meaningful measurement must include the number of harmonics included (and *should* include other information about the test conditions).

**THD+N** means total harmonic distortion plus noise. This measurement is much more common and more comparable between devices. This is usually measured by inputting a sine wave, notch filtering it, and measuring the ratio between the signal with and without the sine wave:

$$\mathrm{THD{+}N} = \frac{\sum \text{harmonic powers} + \text{noise power}}{\text{total output power}}$$

A meaningful measurement must include the bandwidth of measurement. This measurement includes effects from intermodulation distortion, interference, and so on, instead of just harmonic distortion.

## See also

- Audio system measurements

## External links

- Explanation of THD measurements (http://www.dogstar.dantimax.dk/tubestuf/thdconv.htm)

- Rane audio's definition of both THD and THD+N (http://www.rane.com/note145.html)
- Conversion: Distortion attenuation in dB to distortion factor k in % (http://www.sengpielaudio.com/calculator-thd.htm)

Retrieved from "http://en.wikipedia.org/wiki/Total_harmonic_distortion"

Category: Electronics terms

# Application note # 8

## Sample Champion

**NOTE: since Sample Champion Version 3.8, it is possible to measure SNR, THD, THD+N and IMD in RTA Window (more information here).**

**So, the Audio Quality Plugin is no more required and supported (but still available)**

## Measurement of SNR, THD, THD+N and IMD

First of all, select appropriate measurement settings in the "Settings" window. For example use following ones:

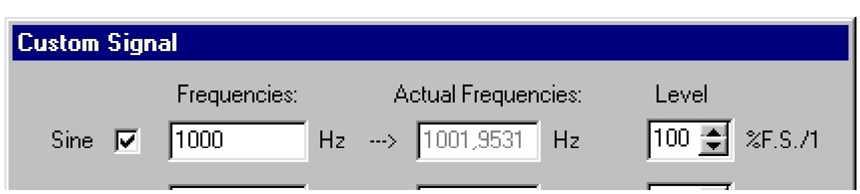
- Sampling Rate: 48000 Hz
- Block: 16
- Mode: Average
- Step: 4
- Custom Signal
- FFT Length 4K
- Weighting Window: Full Blackman-Harris

and connect the audio device under test to the sound card.
In the following examples, the test has been performed using a "loop-back" configuration.

### SNR (Signal to Noise Ratio) measurement

The **SNR** value is the ratio of the peak power level to the remainig noise power.

**Measurement procedure:**
In the Custom Signal Window a single pure tone must be selected. The following figure shows an example (1 kHz tone).



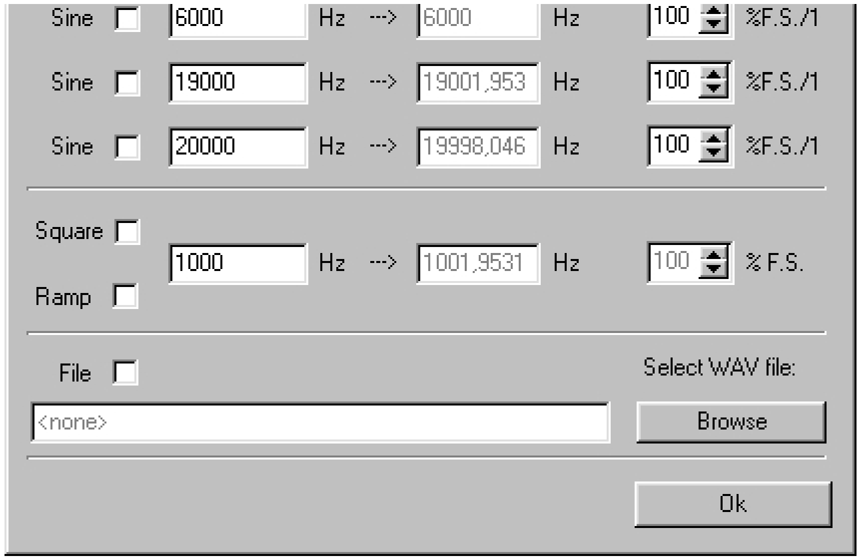



Figure 1 - Single pure tone selection

Open the Scope Window, press the "Select All" button and pull down completely the X Zoom slider. Press the "Syncro Start (REC & PLAY)" button and check the input and output levels. The wave resulting from the measurement should look like the following one (note that all data must be selected by means of the weighting window to achieve an high degree of accuracy):

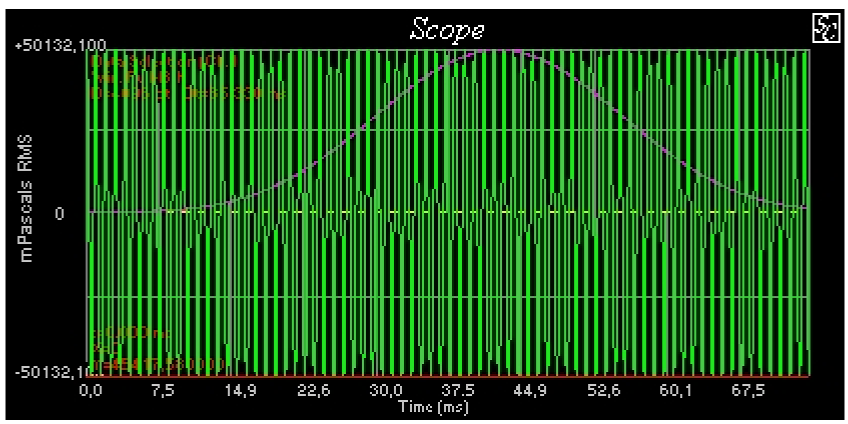
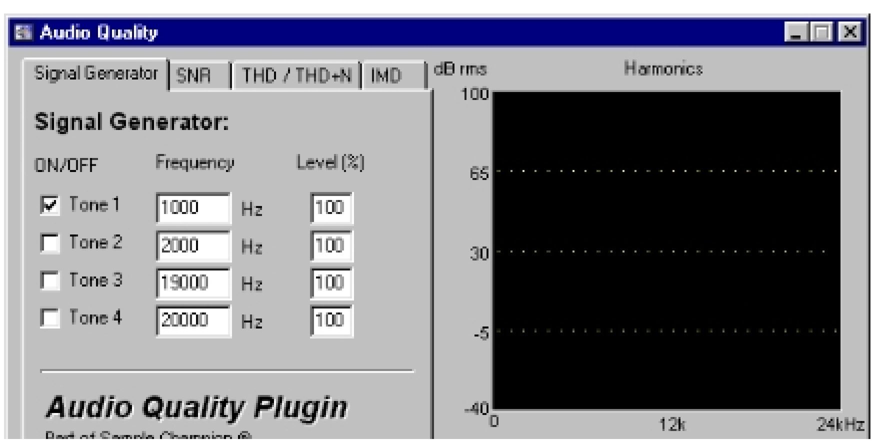



Figure 2 - Sampled 1 Khz pure tone

Now the Audio Quality plugin can be opened (from the "New Measurement" Window).



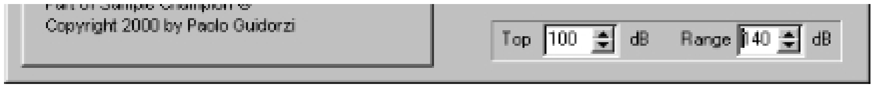



Figure 3 - Audio Quality Plugin

The first page of the plugin shows Custom Generator settings (the frequencies of the signal generator pure tones can be set or modified only from the main program).

In the Audio Quality plugin, select the SNR page and start a new measurement cycle.

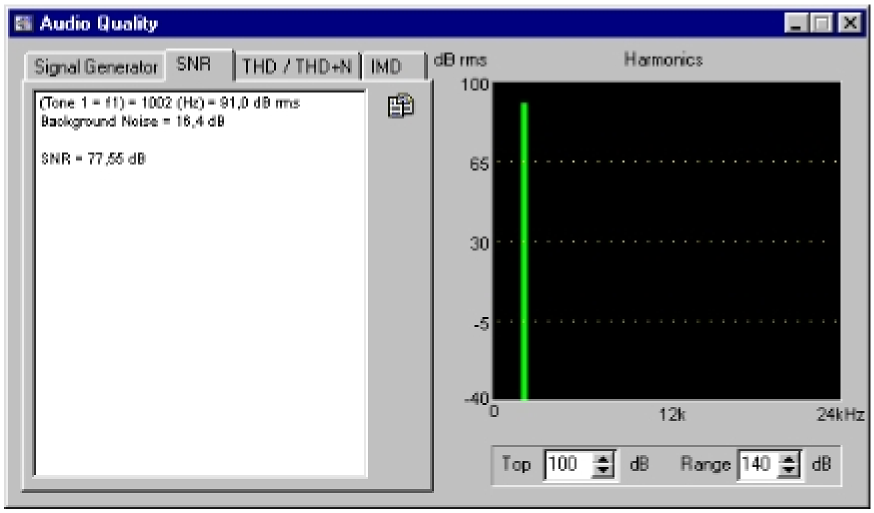



Figure 4 - SNR measurement

The space on the right shows the detected pure tone, represented by a green bar. The space on the left reports the measured value and the computed SNR. In the registered version of the plugin, this text can be copied to the Clipboard.

If the Averaging Mode has been selected, at each cycle the SNR value will decrease until the minimum value is reached.

## THD (Total Harmonic Distortion) measurement

The **THD** is defined by the following formula:

$$\%THD = \frac{\sqrt{H_2^2 + H_3^2 + \ldots + H_N^2}}{\sqrt{H_1^2 + H_2^2 + H_3^2 + \ldots + H_N^2}} \times 100$$

where terms 2..N are the power levels of the harmonics and term 1 is the power level of the fundamental (the pure tone).

The **THD+N** is defined by the following formula:

$$\%THD{+}n = \frac{\sqrt{H_2^2 + H_3^2 + \ldots + H_N^2 + n^2}}{\sqrt{H_1^2 + H_2^2 + H_3^2 + \ldots + H_N^2 + n^2}} \times 100$$

where term n is the noise power level.

**NOTE:** since Sample Champion Version 3.8 and above, it is possible to select a different computation formula of THD and THD+N.

THD and THD+N can now be computed in **RTA window** (see here), so there is no need to use the Audio Quality Plugin.

In **RTA window** is available the *THD Mode* option.


Enabling the THD Mode option, **THD** will be computed by means of the following formula:

$$\%THD = \frac{\sqrt{H_2^2 + H_3^2 + \ldots + H_N^2}}{\sqrt{H_1^2}} \times 100$$

and **THD+N** will be computed by means of the following formula:

$$\%THD{+}n = \frac{\sqrt{H_2^2 + H_3^2 + \ldots + H_N^2 + n^2}}{\sqrt{H_1^2}} \times 100$$

In normal measurements (with low THD and THD+N) the 2 methods will give identical results, since more than 99% of the measured energy is always contained in the fundamental harmonic (H1).

In RTA window is implemented also a new method for the evaluation of the Noise Floor (see here a detailed explanation).

**Measurement procedure:**
In the Custom Signal Window a single pure tone must be selected.
Keep the same selection in Scope Window as in the previous case. In the Audio Quality plugin, select the THD page and start a new measurement cycle.

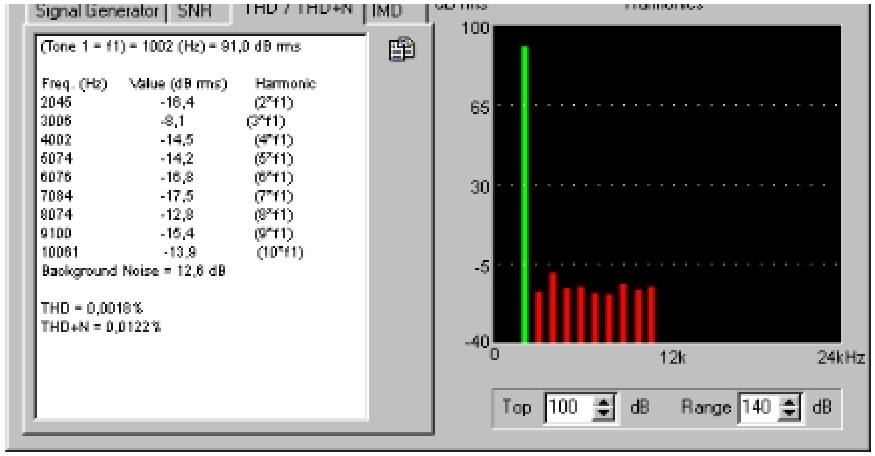



Figure 5 - THD measurement

The space on the right shows the detected pure tone, represented by a green bar and the first 9 harmonics, represented by red bars. The space on the left reports the measured values and the computed THD and THD+N. In the registered version of the plugin, this text can be copied to the Clipboard.

## IMD (InterModulation Distortion) measurement

This parameter gives a measure of the distortion caused in the device under test by two pure tones (cross modulated power). The following harmonics are considered:

(Tone 1 = f1)
(Tone 2 = f2)
(f2-f1)
(f1-2*(f2-f1))
(f1-(f2-f1))
(f1+2*(f2-f1))
(f1+3*(f2-f1))
(2*f1)
(f1+f2)
(2*f2)
(3*f1)
(2*f1+f2)
(2*f2+f1)
(3*f2)

The **IMD** value is computed as the ratio of the sum of the power levels of the intermodulation harmonics to the sum of the power level of the two strongest tones.

**Measurement procedure:**
In the Custom Signal Window two pure tones must be selected. Keep the same selection in Scope Window as in previous cases. In the Audio Quality plugin, select the IMD page and start a new measurement cycle.

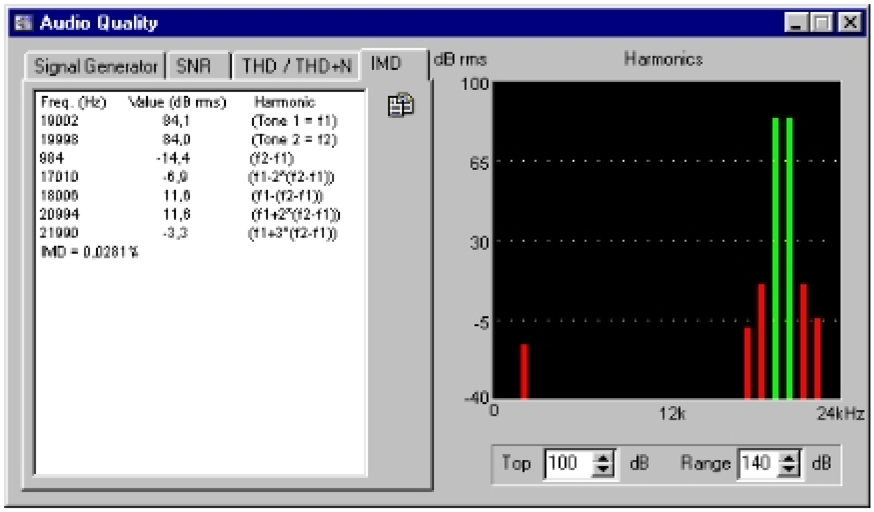



Figure 6- IMD measurement (19 and 20 kHz tones)

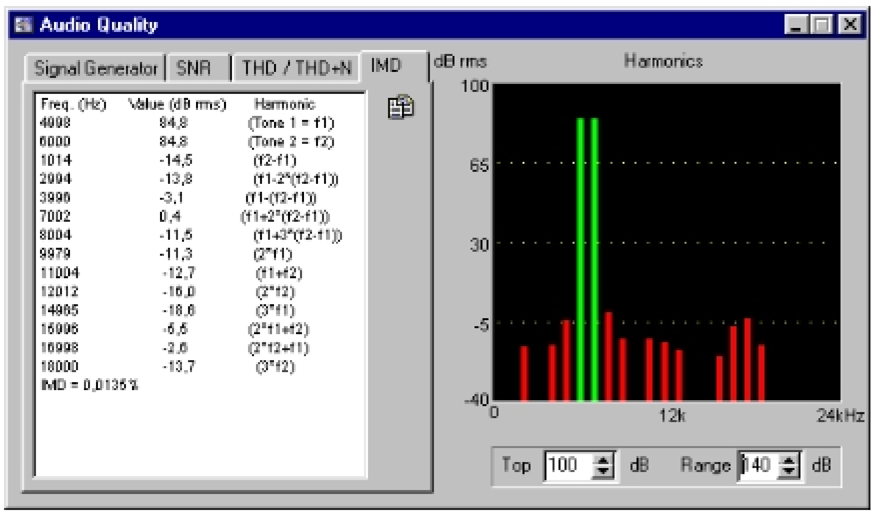



Figure 7- IMD measurement (5 and 6kHz tones)

The space on the right shows the two detected pure tones, represented by green bars and the considered harmonics, represented by red bars. The space on the left reports the measured values and the computed IMD. In the registered version of the plugin, this text can be copied to the Clipboard.

Common choices of the two fundamental frequencies are:

- SMPTE: 60 Hz and 7 kHz (4:1 ratio)
- DIN: 250 Hz and 8 kHz
- CCIF: 19 kHz and 20 kHz

The Audio Quality plugin shows also the narrow-band version of the computed spectrum.

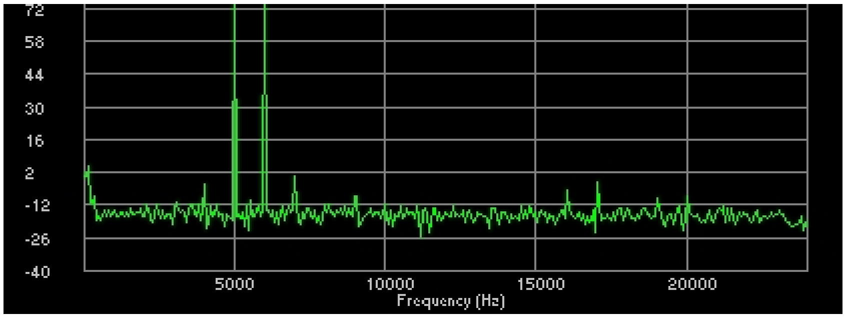



Figure 8- Narrow band spectrum

**NOTE: since Sample Champion Version 3.8, it is possible to measure SNR, THD, THD+N and IMD in RTA Window (more information here).**

**So, the Audio Quality Plugin is no more required and supported (but still available)**

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# Example: Violin

- Produces a waveform that is nearly a sawtooth
- Signal has significant distortion
- Sound is pleasurable
- Slight change in wave shape (relative distortion components) would be obnoxious

Amplifiers: Circuits that scale a signal by a constant amount

Ideally: - $V_o(t) = A\,V_i(t)$ where $A$ is a constant

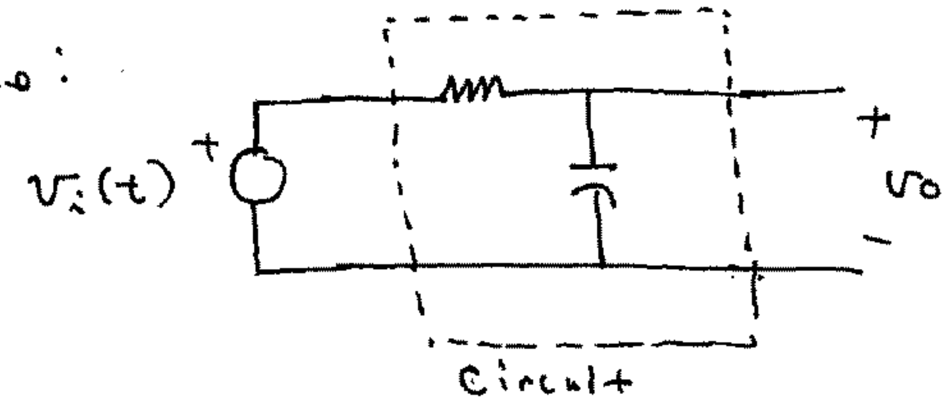
- Linearity is important

- Even small amounts of distortion are objectionable in most applications

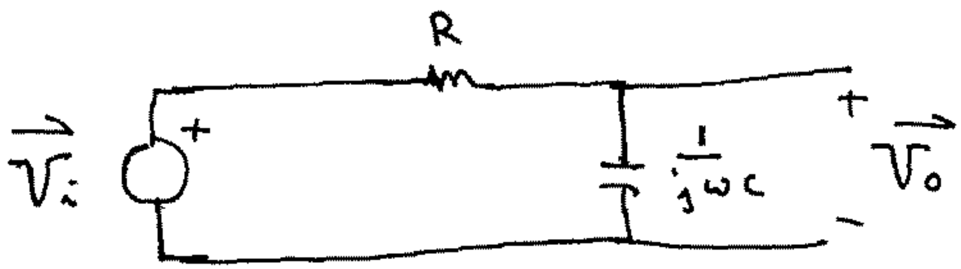
- Frequency distortion also problematic in many applications

- Frequency distortion can be present in linear amplifiers

Example:




If $v_i$ is a sinusoid, the following phasor domain circuit can be drawn

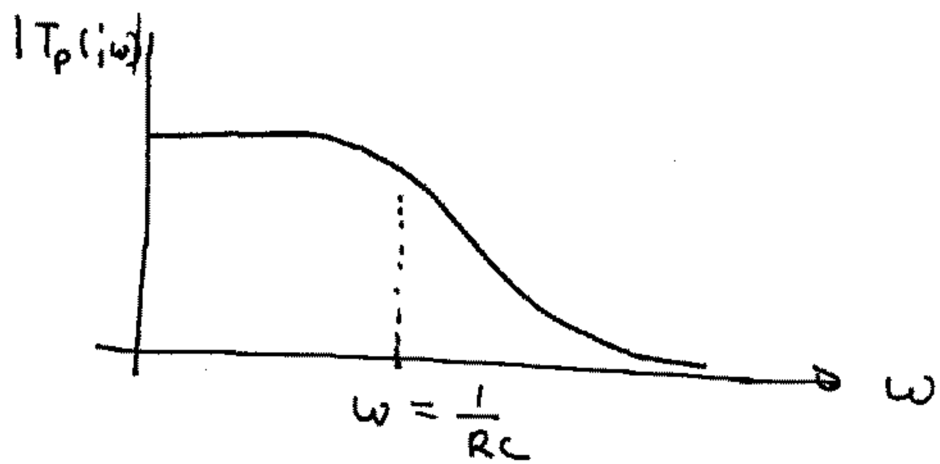



$$\vec{V_o} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \vec{V_i} \Rightarrow \frac{\vec{V_o}}{\vec{V_i}} = \frac{1}{1 + j\omega RC}$$

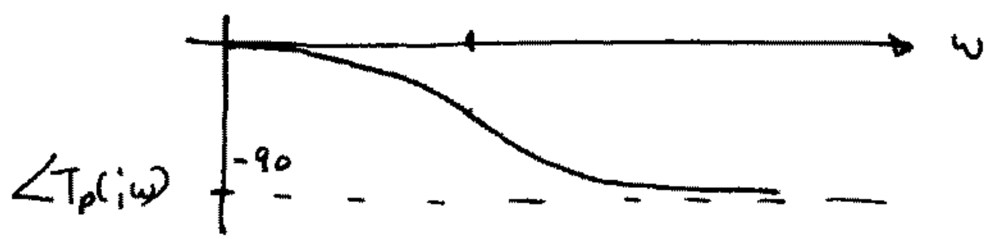
$$\therefore \quad T_p(j\omega) = \frac{1}{1 + j\omega RC}$$

$$T_p(j\omega) = \frac{1}{1 + j\omega RC}$$

$$|T_p(j\omega)| = \frac{1}{\sqrt{1+\omega^2R^2C^2}} \qquad \angle T_p(j\omega) = -\tan^{-1}(\omega RC)$$




Magnitude Plot




Phase Plot

Theorem: If a network is linear, then the steady state response due to a sinusoidal excitation of $V_i = V_m \sin(\omega t + \gamma)$ is

$$V_o(t) = V_m \left| T_p(j\omega) \right| \sin\left(\omega t + \gamma + \angle T(j\omega)\right)$$

- This is a very important theorem and is one of the major reasons phasor analysis was studied in EE201.

- SSS response completely determined by $T_p(j\omega)$

- SSS response can be written by inspection of $|T_p(j\omega)|$ and $\angle T_p(j\omega)$ plots

- Authors of current electronics textbook do not talk about phasors or $T_p(j\omega)$
- This is consistent with the industry when discussing electronic circuits and systems
- The sinusoidal steady state response is of considerable concern in electronic circuits and is used extensively in this text
- Authors & industry use concept of transfer function $T(s)$

End of material covered in Lecture 3

End of material covered in Lecture 3

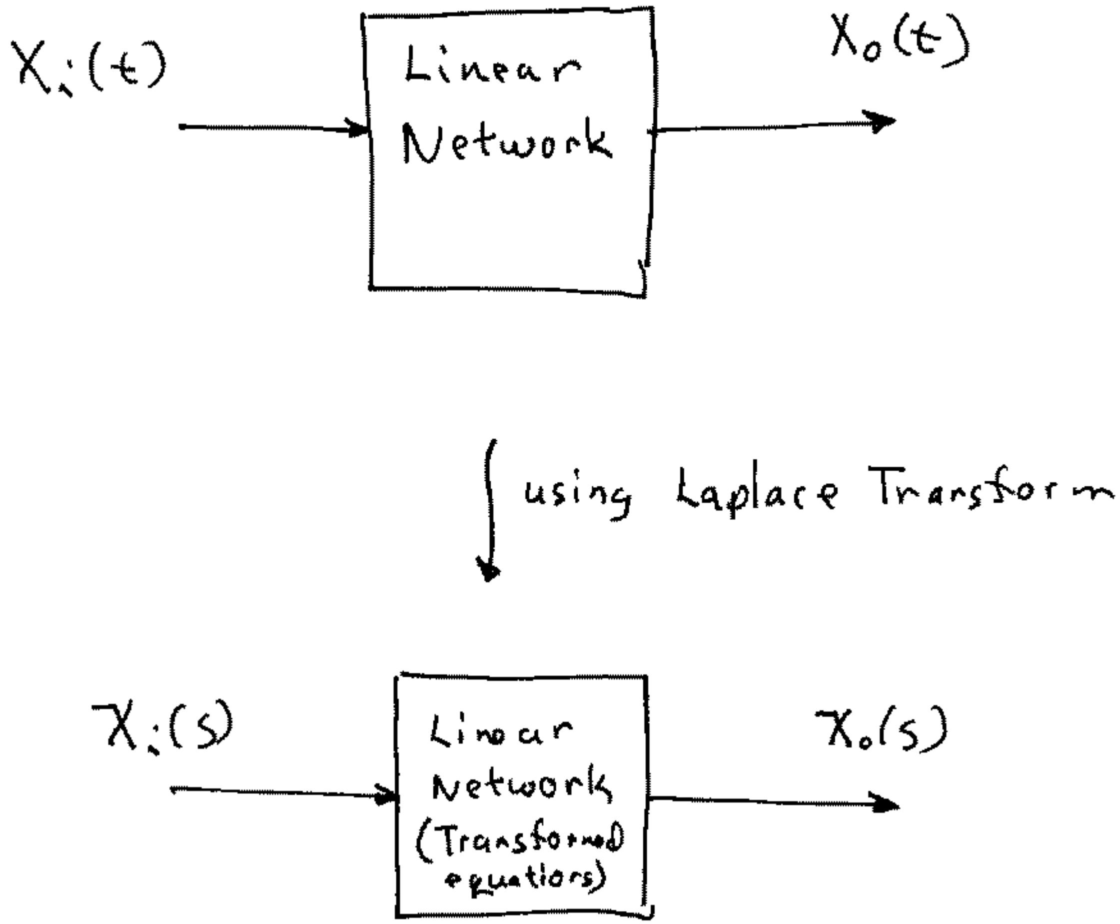
How does $T(s)$ relate to $T_p(j\omega)$ ?

why is $T(s)$ used instead of $T_p(j\omega)$ ?

What is $T(s)$ ?

Why was $T_p(j\omega)$ used in EE201 instead of $T(s)$ for characterizing frequency dependence of linear networks ?

What is $T(s)$?




$$\frac{X_o(s)}{X_i(s)} = T(s)$$

what is $T(s)$ ?

$T(s)$ is the ratio of the Laplace Transform of the output to the Laplace Transform of the input

$T(s)$ is called the transfer function

Theorem: If the input to a linear network with transfer function $T(s)$ is $V_m \sin(\omega t + \gamma)$, then the sinusoidal steady state response is

$$V_o(t) = V_m |T(j\omega)| \sin(\omega t + \gamma + \angle T(j\omega))$$

In the differential equations class, $T(s)$ was obtained by taking the Laplace transform of a set of differential equations of a linear system

Why was the Laplace Transform concept introduced in the differential equations class?

The Laplace Transform of a set of differential equations in the time domain resulted in a set of <u>Linear</u> equations in the transformed domain

The resultant set of linear equations was much easier to solve in most cases than the set of differential equations

$$X_o(t) = \mathcal{L}^{-1} T(s) X_i(s)$$

How does $T(s)$ relate to $T_p(j\omega)$ ?

$$T(s)\Big|_{s=j\omega} = T_p(j\omega)$$

we know how to determine $T_p(j\omega)$. Is there an easy way to obtain $T(s)$ ?